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Discussion

Discussion of "The asymmetry of stress in granular media" [J.P. Bardet and I. Vardoulakis, Int. J. Solids Struct. 2001, Vol. 38, No. 2, pp. 353–367]

Matthew R. Kuhn *

Department of Civil and Environmental Engineering, University of Portland, 5000 N. Willamette Blvd., Portland, OR 97203, USA

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Abstract

The discussion concerns a recently proposed definition of average stress for granular materials, one which can manifest asymmetry in the absence of surface couples, body couples, and contact couples. The average stress was derived from a new postulate that employs virtual work terminology. The discussion shows that the postulate leads to a non-unique average stress and to a non-unique stress asymmetry.

Keywords: Granular mechanics; Couple stress; Micromechanics; Stress asymmetry; Virtual work

The authors present a definition of average stress for granular materials that is derived from a new virtual work postulate. In deriving this stress, the authors account for the finite extent of an averaging region by allowing the boundary to pass through peripheral grains. The authors then select a set of discrete internal forces that are readily identifiable within a granular assembly: the contact forces and moments between pairs of neighboring particles, $\{\mathbf{f}^c, \mathbf{m}^c : c \in I\}$. This set of internal, inter-particle contact forces includes only those contact points $\{\mathbf{x}^c\}$ that lie within the interior region, $c \in I$. The authors also identify a set of discrete external forces and moments, $\{\mathbf{f}^e, \mathbf{m}^e : e \in E\}$, which act upon exterior points $\{\mathbf{x}^e\}$ of peripheral particles that straddle the region's boundary. Notably, the points $\{\mathbf{x}^e\}$ may lie beyond the region. This extension of the concept of *boundary tractions* arises naturally from an acknowledgement of the granular, discrete nature of these materials.

The authors use the principle of virtual work to establish two sets of internal movements, $\{\Delta\mathbf{u}^c\}$ and $\{\Delta\boldsymbol{\theta}^c\}$, that are the duals of the internal (contact) forces, $\{\mathbf{f}^c, \mathbf{m}^c : c \in I\}$:

$$\Delta\delta u_i^c = \delta u_i^b - \delta u_i^a + e_{ijk}(\delta\theta_j^b(x_k^c - x_k^b) - \delta\theta_j^a(x_k^c - x_k^a)) \tag{9_1}$$

* Tel.: +1-503-943-7361; fax: +1-503-943-7316.
E-mail address: kuhn@up.edu (M.R. Kuhn).

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$$\Delta\delta\theta_i^c = \delta\theta_i^b - \delta\theta_i^a \tag{9$_2$}$$

In these equations, $\mathbf{x}^a$ and $\mathbf{x}^b$ are reference points attached to particles "a" and "b", which the authors have placed at the particle centers, and $\mathbf{x}^c$ is the contact point of the particle pair.

In a separate use of virtual work, the authors choose five patterns of virtual displacement, as expressed in the variations a_i, b_{ij}, c_{ijk}, α_i, and β_{ij}. Contributions of virtual work are then separated into two groups: contributions produced by the discrete internal forces $\{\mathbf{f}^c, \mathbf{m}^c : c \in I\}$, and contributions produced by the discrete external forces $\{\mathbf{f}^e, \mathbf{m}^e : e \in E\}$. The sum of contributions in each group, labeled δW_I^D and δW_E^D, must obey the equilibrium condition

$$\delta W_I^D + \delta W_E^D = 0 \tag{7}$$

in which superscript D denotes a discrete media. Condition (7) must be satisfied by any combination of the five variations. For example, the authors show that variations b_{ij} and α_i lead to the following two equilibrium conditions:

$$\sum_{c\in I}(x_i^b - x_i^a)f_j^c = \sum_{e\in E} x_i^{ae} f_j^e, \quad i,j = 1,2,3 \tag{17}$$

$$\sum_{c\in I} e_{ijk}(x_j^b - x_j^a)f_k^c = -\sum_{e\in E} M_i^{ae}, \quad i = 1,2,3 \tag{18}$$

Point $\mathbf{x}^{ae}$ is the reference point of a particle "a" that straddles the boundary (i.e. a peripheral particle), and some of the reference points $\mathbf{x}^a$ and $\mathbf{x}^b$ may also belong to peripheral particles. Vector $\mathbf{M}^{ae}$ is the external moment acting on a peripheral particle "a", with the moment taken about the particle's reference point $\mathbf{x}^{ae}$:

$$M_i^{ae} = e_{ijk}(x_j^e - x_j^{ae})f_k^e + m_i^e \tag{21}$$

The left and right sums in Eqs. (17) and (18) correspond to δW_I^D and $-\delta W_E^D$ in condition (7) for the two variations b_{ij} and α_i. We now arrive at a fundamental observation.

> Although the sum of δW_I^D and δW_E^D must equal zero (as in Eq. (7)), the values of δW_I^D and $-\delta W_E^D$ on the two sides of Eqs. (17) and (18) depend upon the choices of the reference points $\mathbf{x}^a$, $\mathbf{x}^b$, and $\mathbf{x}^{ae}$ that are assigned to the particles. Neither δW_I^D nor δW_E^D is unique. Virtual work can be transferred between δW_I^D and δW_E^D by shifting the reference points of the peripheral particles.

This observation does not invalidate principle (7), which is a statement of equilibrium. The authors, however, use the two (discrete) sums δW_I^D and δW_E^D to define an average stress within a discrete, granular media. The authors arrive at this average stress by comparing these two sums with two other virtual work quantities, δW_I and δW_E, which are separately derived for a *continuous* media. The internal and external forces on the continuous media are stress, couple stress, traction, and moment traction. These continuum forces must obey the equilibrium condition

$$\delta W_I + \delta W_E = 0 \tag{29}$$

The authors propose the following postulate as a means of defining an average stress for the discrete media:

$$\delta W_I^D = \delta W_I \quad \text{and} \quad \delta W_E^D = \delta W_E \tag{43}$$

This postulate and the left side of Eq. (17) lead to the definition of an average stress $\overline{\sigma}_{ij}$ that uses the (discrete) internal forces within the granular media,

$$\overline{\sigma}_{ij} = \frac{1}{V}\sum_{c\in I}(x_i^b - x_i^a)f_j^c, \tag{44$_1$}$$

and the authors show that this stress can be asymmetric, with an asymmetry amplitude derived from Eq. (18),

$$\overline{\sigma}_{ij} - \overline{\sigma}_{ji} = -(e_{ijk} - e_{jik})\frac{1}{V}\sum_{e\in E}M_k^{ae} \tag{47$_2$}$$

Neither the stress $\overline{\sigma}_{ij}$ (44_1) nor its asymmetry (47_2) is unique, since neither δW_I^D nor δW_E^D in (43) is unique. The authors have chosen to place the reference points $\mathbf{x}^a$, $\mathbf{x}^b$, and $\mathbf{x}^{ae}$ at the centers of particles. By shifting the reference points of the peripheral particles (or the reference point of even a single peripheral particle), δW_I^D will change, $\overline{\sigma}_{ij}$ will change, and the stress asymmetry will change.

Postulate (43) is flawed and should not be used to define an average stress.